Let's Face Complexity

New Bridges Between Physical and Social Sciences

— International Summer School —
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SCIENTIFIC PROGRAM

Directors (Scientific Organizing Committee):

Tassos Bountis, Astana, Kazakhstan
Siegfried Grossmann (Honorary Director), Marburg, Germany
Matjaž Perc, Maribor, Slovenia
Marko Robnik, Maribor, Slovenia

Lecturers:

Tassos Bountis, Astana, Kazakhstan, 2 hours
Siegfried Grossmann, Marburg, Germany, 2 hours
Thomas Guhr, Duisburg, Germany, 8 hours
Klaus Mainzer, Munich, Germany, 4 hours
Peter Marsden, Harvard, USA, 4 hours
Matjaž Perc, Maribor, Slovenia, 4 hours
Tomaž Prosen, Ljubljana, Slovenia, 8 hours
Marko Robnik, Maribor, Slovenia, 2 hours
H. Eugene Stanley, Boston, USA, 4 hours

General content of the lectures by the speakers:

Tassos Bountis, Astana (tassosbountis@gmail.com)
Dynamical and statistical complexity in multi-dimensional Hamiltonian systems

Siegfried Grossmann, Marburg (grossmann@physik.uni-marburg.de)
Examples of complexity in fluid dynamics

Thomas Guhr, Duisburg (thomas.guhr@uni-due.de)
Introduction to econophysics with most recent results

Klaus Mainzer, Munich (mainzer@tum.de)
The cause of complexity: A dynamical, computational and philosophical approach: the role of the Principle of Local Activity

Peter Marsden, Harvard (pvm@wjh.harvard.edu)
Complexity in social sciences

Matjaž Perc, Maribor (matjaz.perc@gmail.com)
Mathematical models of human cooperation

Tomaž Prosen, Ljubljana (tomaz.prosen@fmf.uni-lj.si)
Complexity in nonequilibrium many body quantum systems

Marko Robnik, Maribor (robnik@uni-mb.si)
Introduction to complexity in quantum chaos

H. Eugene Stanley, Boston (hes@bu.edu)
Theory of complex networks with applications to real-world problems

Thus the scientific program comprises the following subfields:

- *Complexity in classical dynamics* (Bountis)
- *Quantum chaos and complexity in many body quantum systems* (Prosen, Robnik)
- *Complexity in fluid dynamics* (Grossmann)
- *Complexity in econophysics* (Guhr)
- *Complexity in social sciences* (Marsden, Perc)
- *General complex networks and applications* (Stanley)
- *General foundations and principles of complexity in science and engineering* (Mainzer)

INTRODUCTION AND MOTIVATION

Complexity science has been developing rapidly during the past few decades, cutting across traditional scientific boundaries and embracing practically all branches of science, both in the realm of fundamental research as well as in practical applications. In all domains, complex systems are studied through mathematical analysis, advanced computer simulations, and increasingly large quantities of data, thereby stimulating revolutionary scientific breakthroughs. Physics is at the heart of all natural sciences and engineering, and complexity science is in this regard certainly no exception. In fact, complexity science has been one of the most vibrant fields of research that expand the boundaries and invigorate traditional areas of physics. Disciplines such as the physics of social systems, or sociophysics, and econophysics have emerged, and are now widely accepted as an integral part of physics. These new disciplines are of course also founded on principles adhering to the following methodological components: (i) Empirical observations of phenomena, introduction of appropriate measurable quantities, and some measured and observed relations between them, (ii) performing reproducible experiments, and (iii) mathematical modelling, supporting theories which aim to improve predictions beyond traditional approaches and unveiling basic laws that govern complex phenomena.

This Summer School has been conceived as a series of lecture courses on complexity in physics in its broadest sense, including social sciences and economics. The program is thus strongly interdisciplinary and of great interest to the new generation of physicists, who will doubtlessly profit from the scientific program of the school. Researchers from other disciplines, including social sciences and economics, are very welcome and encouraged to attend as well. They will surely enjoy and benefit from learning the interdisciplinary approach of solving today's most important problems in the physical and social sciences, along with getting to know the many new bridges that exist between these disciplines. No previous knowledge of the relevant research fields is assumed beyond a bachelors degree. The lecture courses are well suited for beginners, but will also include references to the most recent scientific results. The potential participants, therefore, are master course students, PhD students, postdocs, and other junior and senior researchers interested in these topics.

It should be emphasized that the speakers will not only give lectures but will also interact with the students to form subgroups of those that are especially interested in their Complexity field and meet with them separately (at some suitably chosen times) to discuss their individual research problems. Also, the students will have the possibility to present their work in short reports (maximum eight talks, based on selection and invitation, 15 minutes each, on Wednesday 6 September afternoon, 4.30 - 6.30 pm).

ABSTRACTS OF THE LECTURE COURSES

Complex dynamics and statistics of 1-dimensional Hamiltonian lattices

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One–dimensional Hamiltonian lattices, i.e. chains of coupled nonlinear oscillators, have served for many decades as an ideal model for studying the onset of chaotic behavior in N–particle conservative systems, as well as the transition from classical to statistical mechanics in the thermodynamic limit, where the total energy E and the number of particles N diverge with $\epsilon = E/N =$ constant. In particular, the famous Fermi–Pasta–Ulam (FPU) model, with harmonic and cubic (or quartic) particle interactions has played a central role, since many of the remarkable phenomena it exhibits are similar to what one finds in many similar Hamiltonian systems. In these lectures, I will focus on one class of such phenomena, which may be called "complex" in the sense that they deviate from what common wisdom expects. This class concerns sufficiently high energies that regular (or quasiperiodic) motion is limited and widespread chaotic regions dominate the $2N$–dimensional phase space. Our first discovery is that all types of chaotic behavior are not qualitatively the same. Indeed, very close to the boundaries of regular motion, where Lyapunov exponents are small and orbits exhibit "stickiness" effects, the statistics of averaged position (or momentum) sums is strongly correlated and probability density functions (pdfs) are *not* described by pure Gaussians, associated with what we call " strong chaos" and Boltzmann Gibbs (BG) statistical mechanics. Instead, the pdfs are well approximated by $q > 1$–Gaussians ($q = 1$ being the pure Gaussian), suggesting that their proper description is *not* through the classical BG entropy S_{BG} but rather via the Tsallis' non–additive (and generally non–extensive) S_q entropy, associated by what one might call "weak chaos"[1]. This phenomenon was first observed in low -dimensional FPU models with nearest neighbour interactions [2], as well two–dimensional maps [3] and even 3–dimensional galaxy models [4].

Two years ago, generalisations of the so–called FPU β–model were studied, introducing in the quartic terms *different ranges of interactions* through a coupling constant that decays as $1/r^{\alpha}, 0 \leq \alpha < \infty$ ($\alpha \to \infty$ corresponds to the original nearest neighbor FPU model) [5]. This led to the remarkable observation that under Long Range Interactions (LRI), $0 \leq \alpha < 1$: (i) *complex dynamics*, in the sense that the maximal Lyapunov exponent for high enough specific energies ϵ *decreases* implying that some type of order is restored, and ii) *complex statistics* arises, whereby the distribution of time-averaged velocities is well approached by a $q > 1$-Gaussian, suggesting that the system is "weakly chaotic". For α small enough, a crossover occurs from Tsallis to BG statistics, allowing us to define a *"phase diagram"* for the system, in a way that the $q = 1$ (BG) behavior dominates in the $lim_{N\to\infty} lim_{t\to\infty}$ ordering, while when the ordering is reversed $q > 1$ behavior prevails. More recently [6], we investigated an FPU β–model with LRI in *both* the quadratic and quartic parts of the potential, introducing two exponents α_1 and α_2 in the respective couplings. We thus discovered that "weak chaos" in the sense of decreasing Lyapunov exponents and q Gaussian pdfs, occurs *only when LRI apply to the quartic part*. More importantly, in that case, we obtained extrapolated values for $q > 1$, as $N \to \infty$, suggesting that, in this limit, Tsallis thermostatistics persists and a BG state is never reached! On the other hand, when *LRI are imposed only on the quadratic part*, "strong chaos" and purely Gaussian pdfs are always obtained.

References

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The (log-)law of the wall
and its hydrodynamic origin

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Laminar flows usually have simple profiles, depending on the geometry of the boundary conditions. E. g. the axial velocity profile of fluid flow through a pipe is simply parabolic. Or, the temperature profile in a Rayleigh-Bénard system is linear; etc. All this can be simply concluded from the Navier-Stokes equations, solved for the respective systems in the laminar case.

If the fluid is more strongly driven, the flow becomes turbulent, implying quite different profiles. E. g. in pipe flow the profile of the axial velocity becomes much flatter in the center range and much steeper near the wall. In Rayleigh-Bénard flow plumes appear, supporting and increasing the conductive thermal transport quite strongly by convection, leading to a strong increase of the heat transport.

In the well studied cases after the transition to turbulence the profiles change to the rather characteristic so called **law of the wall**. Turbulent pipe flow e. g. becomes much steeper near the wall and much flatter in the center connected by a characteristic logarithmic profile.

The Navier-Stokes equations for the velocity flow field $\vec{u}(\vec{x}, t)$, which describe Newtonian viscous fluid flow, are vector partial differential equations with a simple looking but highly nontrivial nonlinearity $(\vec{u} \cdot \vec{grad})\vec{u}$. This is of second order in the velocity field and in addition couples the local flow field gradient with the local flow direction. Except for some special cases of laminar flow the Navier-Stokes equations are tractable only by direct numerical simulations, since the flows of interest are strongly turbulent.

Among the most interesting features of turbulent fluid flow are their time averaged profiles. To find these one time averages the Navier-Stokes PDEs. Recent new insight for pipe flow, Taylor-Couette flow and Rayleigh-Bénard flow and the close analogies between them will be reported in the talk. This will shed new light on the so called "law of the wall", i. e., the Prandtl and von Kármán logarithmic boundary layer profile, and how one can go beyond by introducing a generalized turbulent viscosity to properly describe the Reynolds stress and calculate the profiles. The obtained relations may also serve to measure the properties of the generalized turbulent viscosity.

Averaging the hydrodynamic equations of motion one finds the respective transport current densities J, i.e. J^u for the transport current of u_z-field, J^ω for the transport current of the angular velocity field ω, or J^T for the transport current of the thermal field in a Rayleigh-Bénard system. These various transport current densities J in turn determine characteristic velocity scales u_* for the considered flows and geometries. E. g., the flow velocity in the center of the Princeton super-pipe typically is $U(0) \approx 45$ km/h, implying u_* to be of order 0.4 m/s being ≈ 3 per cent of $U(0)$, i. e., $u_* \approx 1.3$ km/h for $Re \approx 6 \cdot 10^6$.

For example in Taylor-Couette flow between an inner and an outer cylinder with respective radii $r_{i,o}$ it is $u_*^2 r_i^2 = J^\omega$. In plane shear flow one finds $u_*^2 = \nu \partial_z U_x(0)$.

A new method to interpret and understand the profiles in the various flows is to introduce the notion of turbulent viscosity $\nu_{turb} \equiv u_* a g(\rho/a)$, where ρ in pipe flow for example denotes the distance from the center of the pipe, i.e. $0 \le \rho \le a$, with a being the pipe's radius. Thus the function $g(x)$ characterizes the position dependent dimensionless turbulent viscosity of the respective system in the respective geometry.

The dimensionless viscosity $g(x)$ can be determined from measured data and then serves to calculate the profiles of interest. From its shape one in particular can study the effects of the curvature of the walls between which the fluid flows.

For thermal Rayleigh-Bénard flow one obtains a thermal **law of the wall**, well consistent with measured data. Typically for Rayleigh numbers of order 10^{12} one finds $u_* \approx 0.0024$ m/s, a thermal width $\delta_{th}^* \propto 1/u_*$ of 630 μm and a typical temperature fluctuation $\Theta \equiv J^T/u_* \approx 2$ K.

Thus analyzing the dimensionless turbulent viscosity g turns out to be a proper tool to understand the turbulent profiles for various typical flows. If g depends linearly on the (relative) distance ρ/a, this implies just the **law of the wall**. Closer inspection allows to also study deviations.

In the talk this new concept of turbulent viscosity $\nu_{turb} \propto g(position)$ and its detailed shape for various relevant flows is discussed. Its usefulness to understand and interpret various recent measurements is demonstrated.

References and Literature for Further Reading

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Introduction to econophysics

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At first sight, it seems a bit far–fetched that physicists work on economics problems. A closer look, however, reveals that the connection between physics and economics is rather natural — and not even new! Many physicists are surprised to hear that the mathematician Bachelier developed a theory of stochastic processes very similar to the theory of Brownian motion which Einstein put forward in 1905. Bachelier did it in the context of financial instruments, and he was even a bit earlier than Einstein. Moreover, not all physicists know that financial time series were a major motivation for Mandelbrot when he started his work on fractals.

In the last 25 years, the physicists' interest in economic issues grew ever faster, and the term "econophysics" was coined. Econophysics developed into a recognized subject. The driving force was the enormously improved availability of economic data, in particular data from the financial markets. It thus became ever more rewarding to do what theoretical physicists always do: *model building based on empirical information.* Moreover, complex systems moved into the focus of physics research. The economy certainly qualifies as a complex system and poses serious challenges for basic research, important examples are non–stationarity and systemic stability of the financial markets. Simultaneously, economics started to become more quantitative. From a practical viewpoint, the need to quantitatively improve economic risk management in, for example, portfolio optimization, was another compelling force for econophysics.

This course is meant to be an introduction. Aspects of basic research as well as applications are discussed. The presentation starts from scratch, background in economics is helpful, but not needed. The following five topics are covered:

1. Basic Concepts
   We begin with explaining markets, particularly financial markets, efficiency, arbitrage and risk. Price and return distributions are shown. Simple stochastic

processes are constructed and their limitations are discussed

2. Detailed Look at Stock Markets and Trading
The descriptive power of standard stochastic processes is limited, which becomes clear when carefully analyzing empirical stock market data. Concepts such as order book, market and limit orders as well as liquidity are explained. Various correlations in the time series of a given stock are studied. A much deeper understanding of stock market trading is achieved.

3. Financial Correlations and Portfolio Optimization
In addition to the above mentioned correlations, there are also (cross) correlations between different stocks, because the companies depend on each other. Important information about markets can be obtained from them. Furthermore, they have a considerable impact on investments, more precisely on how to choose a portfolio comprising shares of different stocks. Methods to optimize such portfolios are presented. The rôle of a special kind of "noise" is discussed.

4. Non–Stationarity and Market States
Qualitatively, it is plausible that markets can function in different states which emerge and stabilize after dramatic events. The (cross) correlations are used to quantitatively identify and extract such different market states. From a more general viewpoint, this gives a new handle on the non–stationarity of financial markets and therby a much improved way to analyse their time evolution. Particular emphasis is given on the still ongoing financial crisis.

5. Credit Risk
A major reason for the present problems in the world economy was a credit crisis, that is, the failure of many individuals and companies to make promised payments. Models for credit risk are presented and evaluated in detail. It is shown by revealing generic features that the benefit of "diversification" is vastly overestimated. This observation has considerable importance when assessing the systemic stability of the financial markets.

Econophysics already comprises a broad spectrum of activities. As time is limited, some of those will not be touched in these lectures. Nevertheless, the material presented in the course provides an overview of major directions in econophysics research. The field develops quickly, implying that not all of the topics in the course can be found in text books appropriate for a physics audience. Some good text books [1–3] written by physicists are listed below, further literature will be given in the course.

References

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The cause of complexity: A dynamical, computational, and philosophical approach

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The session starts with an introduction of complexity as dynamical concept in mathematical physics and complexity as algorithmic concept in computer and information science. In information dynamics, both concepts are combined and play an important role to model complexity in natural as well as engineering sciences. The principle of local activity explains the emergence of complex patterns in a homogeneous medium. At first defined in the theory of nonlinear electronic circuits in a mathematically rigorous way, it can be generalized and proven at least for the class of nonlinear reaction-diffusion systems in physics, chemistry, biology, and brain research. Recently, it was applied to memristors in Hodgkin-Huxley neurons and nanoelectronic circuits to generate action potentials in neuromorphic architectures of computers and brains. We argue that the principle of local activity is really fundamental in science and can even be identified in applications of economic, financial, and social systems with the emergence of non-equilibrium states, symmetry breaking at critical points of phase transitions, and risk taking at the edge of chaos.

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Lectures for the "Let's Face Complexity"

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1. An overview of social network studies in the social sciences

This talk will outline the history and scope of the study of social networks in the social sciences. It will begin with precursor developments in social psychology (sociometry) and social anthropology, prior to the codification of social networks as a recognized interdisciplinary field in the 1970s and 1980s, followed by its rapid expansion beginning in the 1990s and extending to the present time. It will sketch the defining features of the field in present-day practice, and highlight distinctive features (such as homophily and relatively high clustering/closure) that are characteristic of social as distinct from other types of networks. Finally, it will survey and exemplify the types and range of social science problems studied using network methods.

Selected readings: [1,2]

2. Social networks and achievement

Social scientists have devoted a great deal of attention over the past quarter century to the notion of "social capital". One prominent sociologist, James S. Coleman, defined social capital as some aspect of social structure that "facilitates certain actions of actors" and makes possible "the achievement of certain ends that in its absence would not be possible". Social capital is measured in numerous ways and takes various forms in particular circumstances, but virtually always involves some feature of social networks. It has been associated with various consequences, such as greater interpersonal trust, civic engagement, or access to impacted information. This talk will focus on the local density that is, the extent of closure of an interpersonal network, and its association with individual achievement and advancement. Persons embedded in more open networks are apt to be connected with more diverse segments of a social group, and to have access to a greater variety of information on a more timely basis. This in turn may offer an individual advantages in a competitive arena. Among other things, this talk will discuss typical survey methods for obtaining data about social networks, and certain indices for measuring the structure of egocentric networks.

Selected readings: [3,4]

3. Diffusion and influence via social networks

Social networks are of interest to social scientists largely because they help to interpret/understand the behavior of the individual units or "actors" of which they are composed. An important setting for this is the study of the spread of practices, ideas or pathogens across geographic and social space. This sometimes involves direct contact between change agents possessing a new idea or practice and those "at risk" of adopting it, but it can also involve emulation or imitation at a distance. Some diffusion processes entail simple contagion involving a single contact with someone possessing a practice, while others are density-dependent, requiring that adoptions within someones local social network exceed a certain threshold. One popular account suggests that diffusion is a two-stage process, initiated by innovators and mediated by "opinion leaders" within a group or network. This talk will examine different models and mechanisms of diffusion via social networks, discuss some exemplary applications, and highlight challenges of studying influence and diffusion within social groups.

Selected readings: [5,6]

4. Actor-oriented models of network change and evolution

Historically, social scientists have given comparatively little attention to sources and mechanisms underlying flux and change within social networks. Most studies have had a point-in-time orientation. This is changing rapidly as the availability of longitudinal data on social relationships rises, and as suitable models and estimation techniques for studying network change develop. This talk will introduce one important branch of such work developed by Dutch researchers. It posits that network change emerges from interactions among advantage-seeking actors who initiate and terminate relational ties in an effort to improve their subjectively-defined welfare, e.g. by connecting themselves to more prestigious or similar others, by forming reciprocal relationships and ending non-reciprocal ones, and/or by changing relationships such that a higher level of transitive closure is reached. This actor-level model is embedded within a continuous-time Markov model for network change, and may be estimated from suitable over-time observations on a network via Markov Chain Monte Carlo simulation. An extension of the model examines the coevolution of networks and behavior, allowing an actor-level attitude or behavioral indicator to respond to, as well as shape, network change.

Selected readings: [7,8]

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Mathematical models of human cooperation

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If only the fittest survive, why should one cooperate? Why should one sacrifice personal benefits for the common good? Recent research indicates that a comprehensive answer to such questions requires that we look beyond the individual and focus on the collective behavior that emerges as a result of the interactions among individuals, groups, and societies. Although undoubtedly driven also by culture and cognition, human cooperation is just as well an emergent, collective phenomenon in a complex system. Nonequilibrium statistical physics, in particular the collective behavior of interacting particles near phase transitions, has already been recognized as very valuable for understanding counterintuitive evolutionary outcomes. However, unlike pairwise interactions among particles that typically govern solid-state physics systems, interactions among humans often involve group interactions, and they also involve a larger number of possible states even for the most simplified description of reality.

The human race is remarkable in many ways. We are champions of cooperation [1, 2]. We sacrifice personal benefits for the common good, we work together to achieve what we are unable to achieve alone, we are compassionate, and we are social. And through this cooperation, we have had astonishing evolutionary success. We have conquered our planet, and today there is an abundance of technological breakthroughs and innovations that make our lives better. At the same time, our societies are home to millions that live on the edge of existence. We deny people shelter, we deny people food, and we deny people their survival. We still need to learn how to cooperate better with one another. The problem, however, is that to cooperate more or better, or even to cooperate at all, is in many ways unnatural. Cooperation is costly, and exercising it can weigh heavily on individual wellbeing and prosperity. But why should one perform an altruistic act that is costly to perform but benefits another? Why should we care for and contribute to the public good if freeriders can enjoy the same benefits for free? Since intact cooperation forms the bedrock of our efforts for a sustainable and better future, understanding cooperative behavior in human societies has been declared as one of the grand scientific challenges of the 21st century [3, 4].

While the infusion of statistical physics to this avenue of research is still a relatively recent development [5, 6], evolutionary game theory [7] is long established as the theory of choice for studying the evolution of cooperation among selfish individuals, including humans. Competing strategies vie for survival and reproduction through the maximization of their utilities, which are traditionally assumed to be payoffs that are determined by the definition of the contested game. The most common assumption underlying the evolution in structured populations has been that the more successful strategies are imitated and thus spread based on their success in accruing the highest payoffs. Mutation has also been considered prominently, in that it can reintroduce variation into the population or represent cultural evolution and social learning, in which people imitate those with higher payoffs and sometimes experiment with new strategies. Evolutionary dynamics based on these basic principles has been considered as the main driving force of evolution, reflecting the individual struggle for success and the pressure of natural selection.

Undoubtedly, traditional evolutionary game theory, as briefly outlined above, has provided fundamental models and methods that enable us to study the evolution of cooperation, and research along these lines continues to provide important proof-of-principle models that guide and inspire future research. But the complexity of such systems also requires methods of nonequilibrium statistical physics be used to better understand cooperation in human societies, and to reveal the many hidden mechanisms that promote it.

During the course of the lecture, I will first present the public goods game on the square lattice as the null model of human cooperation [8]. I will then proceed with representative extensions of the game involving punishment [9] and correlated positive and negative reciprocity [10], which deliver the most fascinating examples of phase transitions in the realm of this research. I will conclude with a brief overview of important progress made in other fields, and I will also outline possible directions for future research in the realm of statistical physics of evolutionary games.

As motivation for attending the lectures, I note that by having a firm theoretical grip on human cooperation, we can hope to engineer better social systems and develop more efficient policies for a sustainable and better future.

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Open quantum many-body systems exemplified by the paradigm of the Interacting spin chains out-of-equilibrium: Integrability and complexity

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In my lecture course I will introduce some fundamental concepts in dynamics of strongly interacting quantum (but also classical) lattice systems in one dimension, such as equilibrium and non-equilibrium steady states, local and quasi-local conservation laws, the problem of quantum quench and the steady-state particle/spin/energy transport. Particular attention will be given to a paradigm of a *boundary driven lattice*, where the dynamics in the bulk is generated by *local* and *Hamiltonian* interactions, while the boundary degrees of freedom are coupled to stochastic reservoirs.

In the first part of the lectures, I will introduce canonical markovian description of boundary driven quantum spin chains in terms of the so-called Lindblad equation and discuss some general methods for computing the steady state or the full relaxation dynamics. I will introduce the central concept of the *matrix product ansatz* and demonstrate its versatility for both, analytical and numerical methods. Here I will define the concept of entanglement entropy and discuss the complexity of classical simulations of interacting systems, with examples.

In the second part of the lectures, I will introduce the basic notions of quantum integrability and derive an exact steady state solution of the paradigmatic model: the boundary driven anisotropic Heisenberg spin 1/2 chain (the XXZ model). The concept of quasi-locality shall be introduced and shown how exact steady state solutions can be linked to quasi-local conservation laws. These results have immediate applications to ballistic versus diffusive high temperature quantum transport, and the quantum quench problem of relaxation to a generalized Gibbs state. In turn, the results have also potential practical applications to future quantum technologies, like quantum memories and quantum simulators.

The lecture course will assume only the basic knowledge of quantum and statistical mechanics on the bachelor level.

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Quantum chaos of generic systems

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Quantum chaos (or wave chaos) is a research field in theoretical and experimental physics dealing with the phenomena in the quantum domain (especially regarding solutions of the Schroedinger equation), or in other wave systems, which correspond to the classical chaos. These other wave systems are electromagnetic, acoustic, elastic, surface, seismic, gravitational waves, etc. The classical dynamics describes the "rays" of the underlying waves, and the bridge between the classical and quantum mechanics is the semiclassical mechanics, resting upon the short-wavelength approximations. If the classical dynamics is chaotic, we see clear signatures in the quantum (wave) domain, e.g. in statistical properties of discrete energy spectra, in the structure of eigenfunctions, and in the statistical properties of other observables. Quantum chaos occurs in low-dimensional systems, e.g. with just two degrees of freedom (e.g. in 2D billiards), but of course also in multi-dimensional systems. From the above it is obvious that theory and experiment in quantum chaos are of fundamental importance in physics, and, moreover, also in technology.

In generic Hamilton systems we have regions of stable, regular, motion in the classical phase space for certain initial conditions, and chaotic motion for the complementary initial conditions. Accordingly, the corresponding eigenstates are either regular or chaotic, and also the corresponding energy spectra have different statistical properties, namely either Poissonian for the regular eigenstates or the statistics of random matrices in the chaotic case. In order to decide whether a given eigenstate and the corresponding energy level is regular or chaotic, we must look into the structure of Wigner functions in the "quantum phase space".

Quantum localization of classically chaotic eigenstates is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time t_H of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The

Heisenberg time t_H, as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant $2\pi\hbar$ and the mean spacing between two nearest energy levels ΔE, $t_H = 2\pi\hbar/\Delta E$.

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems, and shall demonstrate the approach and the method in the case of a billiard family in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures, using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure A is defined by means of the information entropy, whilst the second one, C, in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measueres are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution $P(S)$ (of nearest neighbours), which at small S is linear $P(S) \propto S$, and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function $P(S) = \exp(-S)$), where there is no level repulsion ($P(0) = 1 \neq 0$). In fully chaotic regime with quantum localization we observe that $P(S)$ at small S is a power law $P(S) \propto S^\beta$, with $0 < \beta < 1$. We shall show that there is a functional dependence between the localization measure A and the exponent β, namely that β is a monotonic function of A: in the case of the strong localization are A and β small, while in the case of weak localization (almost extended chaotic states) A and β are close to 1. This presentation includes also our very recent papers.

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Introduction to the theory of complex networks with applications to real-world problems

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These lectures will introduce, in pedagogical detail, complex networks—the key experiments, the concepts and the theory. We will also introduce some examples where these concepts are very important. Among the most dramatic examples are disasters—that range from abrupt financial "flash crashes" and large-scale power outages to sudden deaths among the elderly. These exemplify the fact that the most dangerous vulnerabilities are hiding in the many interdependencies among different networks. We will argue that dramatic network phenomena arise often from mechanisms arising in *interconnected* networks and demonstrate the need to consider mutually dependent network properties in designing resilient systems. Specifically, our team has recently uncovered new laws governing the nature of switching phenomena in coupled networks. We found that phenomena that are continuous *second order* phase transitions in isolated networks become discontinuous abrupt *first order* transitions in interdependent networks [J. Gao, S. V. Buldyrev, H. E. Stanley, and S. Havlin, "Novel Behavior of Networks Formed from Interdependent Networks," Nature Physics **8**, 40 (2012)]. We also discuss parallel efforts to understand the phenomenon of spontaneous recovery in dynamical networks as occurs, e.g., immediately after a flash crash [A. Majdandzic, B. Podobnik, S. V. Buldyrev, D. Y. Kenett, S. Havlin, and H. E. Stanley, "Spontaneous Recovery in Dynamical Networks," Nature Physics **10**, 34–38 (2014)]. The work underlying this course was carried out in collaboration with a number of students & colleagues, chief among whom are S. Havlin & R. Parshani (Bar-Ilan), S. V. Buldyrev (Yeshiva U), B. Podobnik, A. Majdandzic, J. Gao & G. Paul (BU), T. Preis & H. S. Moat (Warwick), and is summarized in readable format as a feature cover article "When Networks Network" *Science News* (22 September 2012), based largely on the paper S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, "Catastrophic Cascade of Failures in Interdependent Networks," Nature 464, 1025–1028 (2010).