Let's Face Complexity

New Bridges Between Physical and Social Sciences

— International Summer School —
http://lfcs.lakecomoschool.org/

Villa del Grumello, Como, Italy
4-8 September 2017

Lake Como School of Advanced Studies
http://lakecomoschool.org/

under the general patronage of the

European Academy of Sciences and Arts (Salzburg)

SCIENTIFIC PROGRAM

Directors (Scientific Organizing Committee):

Tassos Bountis, Astana, Kazakhstan
Siegfried Grossmann (Honorary Director), Marburg, Germany
Matjaž Perc, Maribor, Slovenia
Marko Robnik, Maribor, Slovenia

Lecturers:

Michele Armano, ESA, Madrid, Spain, 1 hour
Tassos Bountis, Astana, Kazakhstan, 2 hours
Reuven Cohen, Bar Ilan University, Israel, 6 hours
Thomas Guhr, Duisburg, Germany, 8 hours
Klaus Mainzer, Munich, Germany, 6 hours
Matjaž Perc, Maribor, Slovenia, 4 hours
Tomaž Prosen, Ljubljana, Slovenia, 8 hours
Marko Robnik, Maribor, Slovenia, 2 hours
Günter Schiepek, Salzburg, Austria, 1 hour

General content of the lectures by the speakers:

Michele Armano, ESA, Madrid (michele.armano@esa.int)
LISA Pathfinder Project: An example of a complex physics and engineering system

Tassos Bountis, Astana (tassosbountis@gmail.com)
Dynamical and statistical complexity in multi-dimensional Hamiltonian systems

Reuven Cohen, Bar Ilan University (reuven@math.biu.ac.il)
Introduction to the theory and applications of complex networks

Thomas Guhr, Duisburg (thomas.guhr@uni-due.de)
Introduction to econophysics with most recent results

Klaus Mainzer, Munich (mainzer@tum.de)
The cause of complexity: A dynamical, computational and philosophical approach: the role of the Principle of Local Activity

Matjaž Perc, Maribor (matjaz.perc@gmail.com)
Mathematical models of human cooperation

Tomaž Prosen, Ljubljana (tomaz.prosen@fmf.uni-lj.si)
Complexity in nonequilibrium many body quantum systems

Marko Robnik, Maribor (robnik@uni-mb.si)
Introduction to complexity in quantum chaos

Günter Schiepek, Salzburg (guenter.schiepek@ccsys.de)
Complexity in Human Change Processes

Thus the scientific program comprises the following subfields:

- *Complexity in classical dynamics* (Bountis)
- *Quantum chaos and complexity in many body quantum systems* (Prosen, Robnik)
- *Complexity in econophysics* (Guhr)
- *Complexity in social sciences* (Cohen, Perc)
- *General complex networks and applications* (Cohen)
- *General foundations and principles of complexity in science and engineering* (Mainzer)
- *Complexity in the LISA Project of ESA* (Armano)
- *Complexity in psychology and psychotherapy* (Schiepek)

INTRODUCTION AND MOTIVATION

Complexity science has been developing rapidly during the past few decades, cutting across traditional scientific boundaries and embracing practically all branches of science, both in the realm of fundamental research as well as in practical applications. In all domains, complex systems are studied through mathematical analysis, advanced computer simulations, and increasingly large quantities of data, thereby stimulating revolutionary scientific breakthroughs. Physics is at the heart of all natural sciences and engineering, and complexity science is in this regard certainly no exception. In fact, complexity science has been one of the most vibrant fields of research that expand the boundaries and invigorate traditional areas of physics. Disciplines such as the physics of social systems, or sociophysics, and econophysics have emerged, and are now widely accepted as an integral part of physics. These new disciplines are of course also founded on principles adhering to the following methodological components: (i) Empirical observations of phenomena, introduction of appropriate measurable quantities, and some measured and observed relations between them, (ii) performing reproducible experiments, and (iii) mathematical modelling, supporting theories which aim to improve predictions beyond traditional approaches and unveiling basic laws that govern complex phenomena.

This Summer School has been conceived as a series of lecture courses on complexity in physics in its broadest sense, including social sciences and economics. The program is thus strongly interdisciplinary and of great interest to the new generation of physicists, who will doubtlessly profit from the scientific program of the school. Researchers from other disciplines, including social sciences and economics, are very welcome and encouraged to attend as well. They will surely enjoy and benefit from learning the interdisciplinary approach of solving today's most important problems in the physical and social sciences, along with getting to know the many new bridges that exist between these disciplines. No previous knowledge of the relevant research fields is assumed beyond a bachelors degree. The lecture courses are well suited for beginners, but will also include references to the most recent scientific results. The potential participants, therefore, are master course students, PhD students, postdocs, and other junior and senior researchers interested in these topics.

It should be emphasized that the speakers will not only give lectures but will also interact with the students to form subgroups of those that are especially interested in their Complexity field and meet with them separately (at some suitably chosen times) to discuss their individual research problems. Also, the students will have the possibility to present their work in short reports.

Opening address and foreword

by

SIEGFRIED GROSSMANN

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Dear Professor Marko Robnik!
Dear Co-Directors of the School-Conference:
Professor Tassos Bountis and Professor Matjaž Perc!
Dear lecturers and participants of this year's summer school on complexity and nonlinear dynamics under the title

"Let's face complexity: New bridges between the physical and social sciences"

to be held in Como at Lake Como!

Dear students and colleagues all together!

Allow me please a cordial welcome to all of you, who are participating in the Como-Conference on "Let's face complexity".

This event is part and continuation of a long sequence of Nonlinear Dynamics Conferences, i.e., it is part of a long tradition, which again and again gave new, always exciting stimuli! The series of these chaos conferences started long ago in Ljubljana, Slovenia, really very, very long ago, in 1993, when it was initiated by a group of engaged and very active students, one of them Dr. Maja Malus, now in the US, guided already in the background by the father of what then became a long, a lasting series of events on chaotic dynamics, all organized by Professor Marko Robnik! Thank you really very much, Marko!!

The conference soon changed its location to Professor Robnik's Home-University, i. e. to Maribor, also in Slovenia. We met in a large Hotel. We were all young and strongly engaged in the then new field of physics, namely chaotic, nonlinear dynamics.

Chaos soon turned out to be a wide-spread phenomenon, showing new but nevertheless rather characteristic features as e.g. bifurcation sequences, limited predictability, divergence of trajectories, new types of attractors, statistical time series despite deterministic law by well-defined equations of motion etc., etc.

The present Como-school also is a continuation of the series of schools and conferences organized over the past 30 years by Professor Tassos Bountis at various places in Greece, and we should be grateful to him as well for the great work that he has done for the scientific community.

While in the early days of chaos theory we emphasized the nonlinearity u^2, i. e. the square of a dynamical variable u, later on gradually two main extensions came into our focus: Either higher powers as e.g. u^4 were studied or the field character of the nonlinearity was emphasized, studying $u(x)^2$ or rather, in all hydrodynamic-like examples, the $(\vec{u} \cdot \vec{\nabla})\vec{u}$ type nonlinear interactions, where the nonlinearity not only measures the square of the amplitude u, but its strength also varies with the local slope $\vec{\nabla}$ of the field $u(x)$ together with the directions of the field variable $\vec{u}(x,t)$.

All these generalizations of the nonlinearity enlarged the variety of the phenomena to be treated and it opened a wide and pretty garden of interesting physical or other natural phenomena. Thus nonlinear, chaotic, complex dynamics showed an impressive, highly enriched Pandora's Box of interesting phenomena.

Today nonlinear, complex, chaotic phenomena in nearly all fields of physics and other natural sciences are studied, taking also notice of the field character of the variable. In my eyes, nonlinear, chaotic, complex dynamics has gradually approached real world phenomena, leaving behind the mere model nonlinearities. Meanwhile new aspects of complexity have arisen in practically all branches of science and engineering.

If one looks at the various topics, addressed in this year's "Let's Face Complexity"-meeting, one notices an extremely rich content and mixture of models as well as of real world problems: A really very promising enterprise!

Namely, another characteristic feature of these chaos school-conferences is the richness of the different fields which are met and included. Under the rather global headline "Let's face complexity" a whole plethora of quite different fields is met, increasing amazingly the applicability of the underlying theory in nonlinear dynamics and complexity. This type of interdisciplinarity under the common roof of nonlinear dynamics is another characteristic feature of the school-conferences ”Let’s face chaos through nonlinear dynamics” and of their continuation ”Let’s face complexity”.

Last not least, a completely new stage has been opened this year: The School-Conference ”Let’s face complexity” has moved from the lovely, beautiful city of Maribor and its Piramida-Hotel to another lovely place, to Lake Como. I.e., it has moved from Slovenia to Italy. Now there is competition between two differently located but equal in charm stages, on which you may face nonlinear dynamics and complexity science!

I am absolutely sure that Como at Lake Como in Italy will be as wonderful and as charming a stage for nonlinearity and complexity as Maribor in Slovenia used to be. You will not only experience stimulating and exciting science but also a lovely place. Enjoy both!

To finalize my Skype-mediated introduction to the School-Conference ”Let’s Face Complexity”, I cordially wish you and us a very successful and highly stimulating meeting under the deserving title ”Let’s Face Complexity”. And let me deeply thank Professor Marko Robnik for all his immense work, for his outstanding engagement to organize and realize this wonderful event, and for making everything as perfect as all previous School-Conferences, not only scientifically but also including an outstanding, a fascinating social program, all that at an outstanding location.

I wish I could participate in the Como School-Conference too!

Have a good start and a stimulating first session as well as many further ones! With all my very best wishes I say: ”Good luck”!

In closing, with handover to the first chairman, I say:
”Thank you all for listening”!
And now: ”Como-conference go ahead”!

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SCHEDULE OF ALL TALKS AND ACTIVITIES

Monday, 4 September	
chair	Robnik
09:00-09:15	opening, Grossmann
09:15-10:45	Mainzer
10:45-11:15	coffee & tea
11:15-13:00	Mainzer
13:00-14:15	lunch
chair	Bountis
14:15-16:00	Cohen
16:00-16:30	coffee & tea
16:30-18:00	Cohen
18:00-18:15	Ferčec

Tuesday, 5 September	
chair	Prosen
09:00-10:45	Mainzer
10:45-11:15	coffee & tea
11:15-13:00	Cohen
13:00-14:15	lunch
chair	Guhr
14:15-15:45	Robnik
15:45-16:00	Tang
16:00-16:30	coffee & tea
16:30-18:00	Bountis
18:00-18:15	Vidmar

Wednesday, 6 September	
chair	Cohen
09:00-10:45	Perc
10:45-11:15	coffee & tea
11:15-13:00	Perc
13:00-14:15	lunch
chair	Mainzer
14:15-15:00	Schiepek
15:00-15:45	Armano
15:45-16:15	coffee & tea
16:15-16:30	Bagashov
16:30-16:45	Diaz-Ruelas
16:45-17:00	Lozej
17:00-17:15	Hu
17:15-17:30	break
17:30-17:45	Kaklamanos
17:45-18:00	Kukuljan
18:00-18:15	Rahman
18:15-18:30	Schreiber
18:30-19:00	break
19:00-20:00	concert Goručan
20:00-23:00	conference dinner

Thursday, 7 September	
chair	Perc
09:00-10:45	Prosen
10:45-11:15	coffee & tea
11:15-13:00	Prosen
13:00-14:15	lunch
chair	Cohen
14:15-16:00	Guhr
16:00-16:30	coffee & tea
16:30-18:00	Guhr

Friday, 8 September	
chair	Bountis
09:00-10:45	Guhr
10:45-11:15	coffee & tea
11:15-13:00	Guhr
13:00-14:15	lunch
chair	Mainzer
14:15-16:00	Prosen
16:00-16:30	coffee & tea
16:30-18:00	Prosen
18:00-18:30	closing

ABSTRACTS OF THE LECTURE COURSES

LISA Pathfinder: the complexity of free fall

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(for the LPF collaboration)

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After the thrilling discovery of Gravitational Waves (GW) by the LIGO collaboration on September 14th 2015, the case of taking GW astrometry to space has become even more compelling than before.

Timely, the ESA spacecraft LISA Pathfinder (LPF) was launched on December 3rd 2015 with the mission of testing core technologies to build GW observatories in space.

LPF reached its operational orbit roughly 1.5 million km from Earth towards the Sun in late January 2016. There, it did enable its core instrument, the LISA Technology Package (LTP) - made of a pair of identical gold-platinum cubes, measuring 46 mm and with a mass of 2 kg each - to reach the most precise free fall ever obtained. The spacecraft isolates the cubes at best from all external forces and the sensors design (an ultra-precise interferometer and a capacitors setup around each mass to ensure control) minimize the internal disturbances, while allowing characterization of the instrument via controlled induction of different physical signals. LPF has also hosted a National Aeronautics Space Administration (NASA) companion to the LTP onboard, the Disturbance Reduction System, whose operations took roughly half of the mission allotted time.

The LTP cubes were put in free-fall during the very successful payload commissioning phase, and a set of dedicated experiment have been run onboard ever since and around the clock to characterize the payload performance, both in the nominal and extension phases, until end of June 2017.

LPF was shut down on July 18th 2017 with an impressive score of successes we shall report upon in our presentation: from quality of free fall, to micro-thruster performance, to controllers characterization and global platform reliability, LPF has ticked all the boxes contributing to the solid scientific ground behind the approval of the ESA LISA mission by the Science Programme Committee (SPC) in June 2017, with a foreseen launch date around 2030.

Following this important fact, we shall show how a powerful space-borne GW observatory can be built with LPF technology and a performance close to that originally foreseen for LISA.

Altogether, we shall detail the amazing complexity of the machine and the various steps of the mission, giving relevance to the experience of scientific operations at ESA.

References

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Complex dynamics and statistics of 1-dimensional Hamiltonian lattices

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One–dimensional Hamiltonian lattices, i.e. chains of coupled nonlinear oscillators, have served for many decades as an ideal model for studying the onset of chaotic behavior in N–particle conservative systems, as well as the transition from classical to statistical mechanics in the thermodynamic limit, where the total energy E and the number of particles N diverge with $\epsilon = E/N =$ constant. In particular, the famous Fermi–Pasta–Ulam (FPU) model, with harmonic and cubic (or quartic) particle interactions has played a central role, since many of the remarkable phenomena it exhibits are similar to what one finds in many similar Hamiltonian systems. In these lectures, I will focus on one class of such phenomena, which may be called "complex" in the sense that they deviate from what common wisdom expects. This class concerns sufficiently high energies that regular (or quasiperiodic) motion is limited and widespread chaotic regions dominate the $2N$–dimensional phase space. Our first discovery is that all types of chaotic behavior are not qualitatively the same. Indeed, very close to the boundaries of regular motion, where Lyapunov exponents are small and orbits exhibit "stickiness" effects, the statistics of averaged position (or momentum) sums is strongly correlated and probability density functions (pdfs) are *not* described by pure Gaussians, associated with what we call " strong chaos" and Boltzmann Gibbs (BG) statistical mechanics. Instead, the pdfs are well approximated by $q > 1$–Gaussians ($q = 1$ being the pure Gaussian), suggesting that their proper description is *not* through the classical BG entropy S_{BG} but rather via the Tsallis' non–additive (and generally non–extensive) S_q entropy, associated by what one might call "weak chaos"[1]. This phenomenon was first observed in low -dimensional FPU models with nearest neighbour interactions [2], as well two–dimensional maps [3] and even 3–dimensional galaxy models [4].

Two years ago, generalisations of the so–called FPU β–model were studied, introducing in the quartic terms *different ranges of interactions* through a coupling constant that decays as $1/r^{\alpha}, 0 \leq \alpha < \infty$ ($\alpha \to \infty$ corresponds to the original nearest neighbor FPU model) [5]. This led to the remarkable observation that under Long Range Interactions (LRI), $0 \leq \alpha < 1$: (i) *complex dynamics*, in the sense that the maximal Lyapunov exponent for high enough specific energies ϵ *decreases* implying that some type of order is restored, and ii) *complex statistics* arises, whereby the distribution of time-averaged velocities is well approached by a $q > 1$-Gaussian, suggesting that the system is "weakly chaotic". For α small enough, a crossover occurs from Tsallis to BG statistics, allowing us to define a *"phase diagram"* for the system, in a way that the $q = 1$ (BG) behavior dominates in the $lim_{N\to\infty} lim_{t\to\infty}$ ordering, while when the ordering is reversed $q > 1$ behavior prevails. More recently [6], we investigated an FPU β–model with LRI in *both* the quadratic and quartic parts of the potential, introducing two exponents α_1 and α_2 in the respective couplings. We thus discovered that "weak chaos" in the sense of decreasing Lyapunov exponents and q Gaussian pdfs, occurs *only when LRI apply to the quartic part*. More importantly, in that case, we obtained extrapolated values for $q > 1$, as $N \to \infty$, suggesting that, in this limit, Tsallis thermostatistics persists and a BG state is never reached! On the other hand, when *LRI are imposed only on the quadratic part*, "strong chaos" and purely Gaussian pdfs are always obtained.

References

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Introduction to the theory and applications of complex networks

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The theory of complex networks has attracted considerable attention during the last two decades. With the advancement of computers and the development of the Internet, data from many real world networks started accumulating and analysis of this data became accessible. This lead to theoretical inquiry into the structure, function and dynamics of complex networks.

We will start by a quick introduction to graph theory and random graphs, giving the basic notions and results needed to describe complex networks. We will present tools such as generating functions and branching processes and use them to derive and discuss results of the topological structure and the robustness of random complex networks. We will present other methods such as spectral graph theory and clustering algorithms and show how they can be used to study and understand real world networks. We will discuss the recently introduced notion of "networks of networks", explain its importance, and use mathematical methods to derive results on the robustness and properties of networks of networks.

We will then turn to study some methods in social network analysis and dynamics. We will present some methods for network clustering, allowing identification of different social groups in networks. We will discuss identification of central and influential nodes in a network, and also the dynamics of opinion spreading and consensus formation in networks.

References

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Introduction to econophysics

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At first sight, it seems a bit far–fetched that physicists work on economics problems. A closer look, however, reveals that the connection between physics and economics is rather natural — and not even new! Many physicists are surprised to hear that the mathematician Bachelier developed a theory of stochastic processes very similar to the theory of Brownian motion which Einstein put forward in 1905. Bachelier did it in the context of financial instruments, and he was even a bit earlier than Einstein. Moreover, not all physicists know that financial time series were a major motivation for Mandelbrot when he started his work on fractals.

In the last 25 years, the physicists' interest in economic issues grew ever faster, and the term "econophysics" was coined. Econophysics developed into a recognized subject. The driving force was the enormously improved availability of economic data, in particular data from the financial markets. It thus became ever more rewarding to do what theoretical physicists always do: *model building based on empirical information.* Moreover, complex systems moved into the focus of physics research. The economy certainly qualifies as a complex system and poses serious challenges for basic research, important examples are non–stationarity and systemic stability of the financial markets. Simultaneously, economics started to become more quantitative. From a practical viewpoint, the need to quantitatively improve economic risk management in, for example, portfolio optimization, was another compelling force for econophysics.

This course is meant to be an introduction. Aspects of basic research as well as applications are discussed. The presentation starts from scratch, background in economics is helpful, but not needed. The following five topics are covered:

1. Basic Concepts
   We begin with explaining markets, particularly financial markets, efficiency, arbitrage and risk. Price and return distributions are shown. Simple stochastic

processes are constructed and their limitations are discussed

2. Detailed Look at Stock Markets and Trading
   The descriptive power of standard stochastic processes is limited, which becomes clear when carefully analyzing empirical stock market data. Concepts such as order book, market and limit orders as well as liquidity are explained. Various correlations in the time series of a given stock are studied. A much deeper understanding of stock market trading is achieved.

3. Financial Correlations and Portfolio Optimization
   In addition to the above mentioned correlations, there are also (cross) correlations between different stocks, because the companies depend on each other. Important information about markets can be obtained from them. Furthermore, they have a considerable impact on investments, more precisely on how to choose a portfolio comprising shares of different stocks. Methods to optimize such portfolios are presented. The rôle of a special kind of "noise" is discussed.

4. Non–Stationarity and Market States
   Qualitatively, it is plausible that markets can function in different states which emerge and stabilize after dramatic events. The (cross) correlations are used to quantitatively identify and extract such different market states. From a more general viewpoint, this gives a new handle on the non–stationarity of financial markets and therby a much improved way to analyse their time evolution. Particular emphasis is given on the still ongoing financial crisis.

5. Credit Risk
   A major reason for the present problems in the world economy was a credit crisis, that is, the failure of many individuals and companies to make promised payments. Models for credit risk are presented and evaluated in detail. It is shown by revealing generic features that the benefit of "diversification" is vastly overestimated. This observation has considerable importance when assessing the systemic stability of the financial markets.

Econophysics already comprises a broad spectrum of activities. As time is limited, some of those will not be touched in these lectures. Nevertheless, the material presented in the course provides an overview of major directions in econophysics research. The field develops quickly, implying that not all of the topics in the course can be found in text books appropriate for a physics audience. Some good text books [1–3] written by physicists are listed below, further literature will be given in the course.

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The cause of complexity: A dynamical, computational, and philosophical approach

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The session starts with an introduction of complexity as dynamical concept in mathematical physics and complexity as algorithmic concept in computer and information science. In information dynamics, both concepts are combined and play an important role to model complexity in natural as well as engineering sciences up to machine learning and complex neural networks.

The principle of local activity explains the emergence of complex patterns in a homogeneous medium. At first defined in the theory of nonlinear electronic circuits in a mathematically rigorous way, it can be generalized and proven at least for the class of nonlinear reaction-diffusion systems in physics, chemistry, biology, and brain research. Recently, it was applied to memristors in Hodgkin-Huxley neurons and nanoelectronic circuits to generate action potentials in neuromorphic architectures of computers and brains.

We argue that the principle of local activity is really fundamental in science and can even be identified in applications of economic, financial, and social systems with the emergence of non-equilibrium states, symmetry breaking at critical points of phase transitions, and risk taking at the edge of chaos. Machine learning and complex neural networks become an exciting challenge in the age of digitalization and artificial intelligence. Machine learning and artificial intelligence with their technical and societal impact will be a special issue of the session.

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Mathematical models of human cooperation

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If only the fittest survive, why should one cooperate? Why should one sacrifice personal benefits for the common good? Recent research indicates that a comprehensive answer to such questions requires that we look beyond the individual and focus on the collective behavior that emerges as a result of the interactions among individuals, groups, and societies. Although undoubtedly driven also by culture and cognition, human cooperation is just as well an emergent, collective phenomenon in a complex system. Nonequilibrium statistical physics, in particular the collective behavior of interacting particles near phase transitions, has already been recognized as very valuable for understanding counterintuitive evolutionary outcomes. However, unlike pairwise interactions among particles that typically govern solid-state physics systems, interactions among humans often involve group interactions, and they also involve a larger number of possible states even for the most simplified description of reality.

The human race is remarkable in many ways. We are champions of cooperation [1, 2]. We sacrifice personal benefits for the common good, we work together to achieve what we are unable to achieve alone, we are compassionate, and we are social. And through this cooperation, we have had astonishing evolutionary success. We have conquered our planet, and today there is an abundance of technological breakthroughs and innovations that make our lives better. At the same time, our societies are home to millions that live on the edge of existence. We deny people shelter, we deny people food, and we deny people their survival. We still need to learn how to cooperate better with one another. The problem, however, is that to cooperate more or better, or even to cooperate at all, is in many ways unnatural. Cooperation is costly, and exercising it can weigh heavily on individual wellbeing and prosperity. But why should one perform an altruistic act that is costly to perform but benefits another? Why should we care for and contribute to the public good if freeriders can enjoy the same benefits for free? Since intact cooperation forms the bedrock of our efforts for a sustainable and better future, understanding cooperative behavior in human societies has been declared as one of the grand scientific challenges of the 21st century [3, 4].

While the infusion of statistical physics to this avenue of research is still a relatively recent development [5, 6], evolutionary game theory [7] is long established as the theory of choice for studying the evolution of cooperation among selfish individuals, including humans. Competing strategies vie for survival and reproduction through the maximization of their utilities, which are traditionally assumed to be payoffs that are determined by the definition of the contested game. The most common assumption underlying the evolution in structured populations has been that the more successful strategies are imitated and thus spread based on their success in accruing the highest payoffs. Mutation has also been considered prominently, in that it can reintroduce variation into the population or represent cultural evolution and social learning, in which people imitate those with higher payoffs and sometimes experiment with new strategies. Evolutionary dynamics based on these basic principles has been considered as the main driving force of evolution, reflecting the individual struggle for success and the pressure of natural selection.

Undoubtedly, traditional evolutionary game theory, as briefly outlined above, has provided fundamental models and methods that enable us to study the evolution of cooperation, and research along these lines continues to provide important proof-of-principle models that guide and inspire future research. But the complexity of such systems also requires methods of nonequilibrium statistical physics be used to better understand cooperation in human societies, and to reveal the many hidden mechanisms that promote it.

During the course of the lecture, I will first present the public goods game on the square lattice as the null model of human cooperation [8]. I will then proceed with representative extensions of the game involving punishment [9] and correlated positive and negative reciprocity [10], which deliver the most fascinating examples of phase transitions in the realm of this research. I will conclude with a brief overview of important progress made in other fields, and I will also outline possible directions for future research in the realm of statistical physics of evolutionary games.

As motivation for attending the lectures, I note that by having a firm theoretical grip on human cooperation, we can hope to engineer better social systems and develop more efficient policies for a sustainable and better future.

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Open quantum many-body systems exemplified by the paradigm of the Interacting spin chains out-of-equilibrium: Integrability and complexity

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In my lecture course I will introduce some fundamental concepts in dynamics of strongly interacting quantum (but also classical) lattice systems in one dimension, such as equilibrium and non-equilibrium steady states, local and quasi-local conservation laws, the problem of quantum quench and the steady-state particle/spin/energy transport. Particular attention will be given to a paradigm of a *boundary driven lattice*, where the dynamics in the bulk is generated by *local* and *Hamiltonian* interactions, while the boundary degrees of freedom are coupled to stochastic reservoirs.

In the first part of the lectures, I will introduce canonical markovian description of boundary driven quantum spin chains in terms of the so-called Lindblad equation and discuss some general methods for computing the steady state or the full relaxation dynamics. I will introduce the central concept of the *matrix product ansatz* and demonstrate its versatility for both, analytical and numerical methods. Here I will define the concept of entanglement entropy and discuss the complexity of classical simulations of interacting systems, with examples.

In the second part of the lectures, I will introduce the basic notions of quantum integrability and derive an exact steady state solution of the paradigmatic model: the boundary driven anisotropic Heisenberg spin 1/2 chain (the XXZ model). The concept of quasi-locality shall be introduced and shown how exact steady state solutions can be linked to quasi-local conservation laws. These results have immediate applications to ballistic versus diffusive high temperature quantum transport, and the quantum quench problem of relaxation to a generalized Gibbs state. In turn, the results have also potential practical applications to future quantum technologies, like quantum memories and quantum simulators.

The lecture course will assume only the basic knowledge of quantum and statistical mechanics on the bachelor level.

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Quantum chaos of generic systems

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Quantum chaos (or wave chaos) is a research field in theoretical and experimental physics dealing with the phenomena in the quantum domain (especially regarding solutions of the Schroedinger equation), or in other wave systems, which correspond to the classical chaos. These other wave systems are electromagnetic, acoustic, elastic, surface, seismic, gravitational waves, etc. The classical dynamics describes the "rays" of the underlying waves, and the bridge between the classical and quantum mechanics is the semiclassical mechanics, resting upon the short-wavelength approximations. If the classical dynamics is chaotic, we see clear signatures in the quantum (wave) domain, e.g. in statistical properties of discrete energy spectra, in the structure of eigenfunctions, and in the statistical properties of other observables. Quantum chaos occurs in low-dimensional systems, e.g. with just two degrees of freedom (e.g. in 2D billiards), but of course also in multi-dimensional systems. From the above it is obvious that theory and experiment in quantum chaos are of fundamental importance in physics, and, moreover, also in technology.

In generic Hamilton systems we have regions of stable, regular, motion in the classical phase space for certain initial conditions, and chaotic motion for the complementary initial conditions. Accordingly, the corresponding eigenstates are either regular or chaotic, and also the corresponding energy spectra have different statistical properties, namely either Poissonian for the regular eigenstates or the statistics of random matrices in the chaotic case. In order to decide whether a given eigenstate and the corresponding energy level is regular or chaotic, we must look into the structure of Wigner functions in the "quantum phase space".

Quantum localization of classically chaotic eigenstates is one of the most important phenomena in quantum chaos, or more generally - wave chaos, along with the characteristic behaviour of statistical properties of the energy spectra. Quantum localization sets in, if the Heisenberg time t_H of the given system is shorter than the classical transport times of the underlying classical system, i.e. when the classical transport is slower than the quantum time resolution of the evolution operator. The

Heisenberg time t_H, as an important characterization of every quantum system, is namely equal to the ratio of the Planck constant $2\pi\hbar$ and the mean spacing between two nearest energy levels ΔE, $t_H = 2\pi\hbar/\Delta E$.

We shall show the functional dependence between the degree of localization and the spectral statistics in autonomous (time independent) systems, in analogy with the kicked rotator, which is the paradigm of the time periodic (Floquet) systems, and shall demonstrate the approach and the method in the case of a billiard family in the dynamical regime between the integrability (circle) and full chaos (cardioid), where we shall extract the chaotic eigenstates. The degree of localization is determined by two localization measures, using the Poincaré Husimi functions (which are the Gaussian smoothed Wigner functions in the Poincaré Birkhoff phase space), which are positive definite and can be treated as quasi-probability densities. The first measure A is defined by means of the information entropy, whilst the second one, C, in terms of the correlations in the phase space of the Poincaré Husimi functions of the eigenstates. Surprisingly, and very satisfactory, the two measueres are linearly related and thus equivalent.

One of the main manifestations of chaos in chaotic eigenstates in absence of the quantum localization is the energy level spacing distribution $P(S)$ (of nearest neighbours), which at small S is linear $P(S) \propto S$, and we speak of the linear level repulsion, while in the integrable systems we have the Poisson statistics (exponential function $P(S) = \exp(-S)$), where there is no level repulsion ($P(0) = 1 \neq 0$). In fully chaotic regime with quantum localization we observe that $P(S)$ at small S is a power law $P(S) \propto S^\beta$, with $0 < \beta < 1$. We shall show that there is a functional dependence between the localization measure A and the exponent β, namely that β is a monotonic function of A: in the case of the strong localization are A and β small, while in the case of weak localization (almost extended chaotic states) A and β are close to 1. This presentation includes also our very recent papers.

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Complexity in Human Change Processes Feedback-driven Change Control and Brain Network Dynamics in Psychotherapy

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Human change processes can be conceptualized by models of self-organization in a complex bio-psycho-social system. Psychotherapy may be seen as the facilitation of the self-organizing process of a patient, undergoing critical instabilities and discontinuous pattern transitions. In consequence, theoretical models of psychotherapy should describe the nonlinear interactions of the involved factors and reproduce the patterns of change (non-stationarity) observed by methods of process monitoring. First steps toward modeling psychotherapy by nonlinear difference equations will be presented. Another consequence is the application of methods of frequent and equidistant process monitoring in routine practice together with real-time procedures of data analysis (Synergetic Navigation System, SNS). The SNS allows for an internet-based data collection (e.g., by tablets or smart-phones) for daily self-ratings of therapy-related cognitions and emotions. Data demonstrate that the compliance to engage in such kind of a fine-meshed ambulatory assessment technology is given and feedback-driven practice can enhance therapy outcome. Time series analysis shows features of nonlinear dynamics, non-stationarity, and cascades of critical instabilities (phase-transition like phenomena) in the course of psychotherapies. Strongly connected and synchronized to the mental level, self-organization should also take place at the brain level. In a study with patients suffering from obsessive-compulsive disorder (OCD), repeated fMRI scans were realized during a period of inpatient psychotherapy (2-4 months). The stimulation paradigm used neutral, disgusting, and individually recorded OCD-symptom-provoking pictures to test the changing activation of OCD-relevant brain networks. Considerably larger neural changes in OCD-related brain areas (cingulate cortex/supplementary motor cortex, bilateral dorsolateral prefrontal cortex, bilateral insula, bilateral parietal cortex, cuneus) were observed during critical phases (critical instabilities and pattern

transitions) than during non-critical phases of the psychotherapy or compared to the interscan-intervals of healthy controls. An actual study extended the paradigm concerning the number of fMRI scans during hospital stay and by including a resting state period of 10 minutes at each fMRI scan. Changes in the effective connectivity of neural networks were analysed by the hypothesis-driven method of Dynamic Causal Modeling. The analysis of functional connectivity patterns and their changes refers to a full brain model of 66 cortical and subcortical structures (structural connectivity data from Diffusion Tensor Imaging, DTI) and cross-correlates all functional connectivity (FC[t]) matrices produced by a running window over each resting state period. By this method of Functional Connectivity Dynamics matrices, the non-stationarity of brain dynamics (pattern transitions) is reflected within each resting state, and at another time scale also between resting states realized in the fMRI scans during the course of the therapies. As synergetics predicts, phase-transition like phenomena (acting as non-stationarities) play a crucial role in the psychotherapeutic process supporting self-organisation and complexity models of human change processes.

ABSTRACTS OF THE SHORT REPORTS

Non-perturbative effects in QCD: quantum informational approach

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Perhaps one of the most remarkable problems of the strong interaction physics is the non-perturbative character of interactions at small momentum transfer (large distances and time intervals), which gives rise to the confinement of quarks and gluons and related phenomena. In the present work one of the alternative ways of interpreting those phenomena is presented.

It is possible to consider the model of the stochastic vacuum of QCD (where only the second order field correlators are important) as an environment for colour particles (quarks and gluons). The interactions of colour charged particles with such medium might then be treated in terms of quantum information theory, where analogous interactions lead to the phenomenon of decoherence of colour states of quarks, which turns them in the non-perturbative limit (corresponding to infinitely big Wilson loop area) into fully mixed colour states. This might be considered as an effective generation of confinement, when the pure colour state of quark evolves into a colour mixture with equal probability for each colour as a result of an interaction with the stochastic vacuum.

The description of this process is possible with the help of characteristics used in quantum information theory and quantum optics: purity, fidelity, von Neumann entropy. It might be shown that the decoherence during the interaction of quarks with the stochastic vacuum leads to the loss of quantum information on the initial quark colour, which signals the onset of the confinement. The given formalism also allows to study the intermediate case outside the trivial asymptotic cases of infinitely big and infinitely small transferred momenta.

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Reduction of degrees of freedom in complex systems

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It is well known that low-dimensional nonlinear deterministic maps close to a tangent bifurcation exhibit intermittency and this circumstance has been exploited, e.g., by Procaccia and Schuster [Phys. Rev. A 28, 1210 (1983)], to develop a general theory of 1/f spectra. This suggests it is interesting to study the extent to which the behavior of a high-dimensional stochastic system can be described by such tangent maps. The Tangled Nature (TaNa) Model of evolutionary ecology is an ideal candidate for such a study, a significant model as it is capable of reproducing a broad range of the phenomenology of macroevolution and ecosystems. The TaNa model exhibits strong intermittency reminiscent of punctuated equilibrium and, like the fossil record of mass extinction, the intermittency in the model is found to be non-stationary, a feature typical of many complex systems. We derive a mean-field version for the evolution of the likelihood function controlling the reproduction of species and find a local map close to tangency. This mean-field map, by our own local approximation, is able to describe qualitatively only one episode of the intermittent dynamics of the full TaNa model. To complement this result, we construct a complete nonlinear dynamical system model consisting of successive tangent bifurcations that generates time evolution patterns resembling those of the full TaNa model in macroscopic scales. The switch from one tangent bifur- cation to the next in the sequences produced in this model is stochastic in nature, based on criteria obtained from the local mean-field approximation, and capable of imitating the changing set of types of species and total population in the TaNa model. The model combines full deterministic dynamics with instantaneous parameter random jumps at stochastically drawn times. In spite of the limitations of our approach, which entails a drastic collapse of degrees of freedom, the description of a high-dimensional model system in terms of a low-dimensional one appears to be illuminating

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Integrability of planar polynomial systems of ODE's

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The integrability problem for systems of differential equations is one of the main problems in the qualitative theory of differential systems. In fact, integrability, although a rare phenomenon, is of great importance due to applications in the bifurcation theory. In the study of mathematical models it is important to detect rare systems that are integrable, since perturbations of such systems exhibit a rich behavior of bifurcations.

We consider systems of the form

$$\begin{aligned}\dot{x} &= x - \sum_{i+j=1}^{n-1} a_{i,j} x^{i+1} y^{j} = x - P(x,y)\\ \dot{y} &= -y + \sum_{i+j=1}^{n-1} b_{j,i} x^{j} y^{i+1} = -y + Q(x,y),\end{aligned}$$

where $i \geq -1$, $j \geq 0$.

In this talk we discuss some mechanisms for proving the integrability of polynomial systems. Then we show how these mechanisms can be applied to some polynomial families.

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Periodic solutions of a neutral impulsive differential equation

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The predator-prey model plays a great role in the mathematical ecology. According to the functional response of predator to prey density and its different role in modeling, Holling proposed three types of *monotonic* functional responses $g(x) = x$, $\frac{x}{a+x}$, $\frac{x^2}{a+x^2}$ and a *nonmonotonic* response, so-called *Holling type IV* functional response $g(x) = \frac{x}{a+x+\frac{x^2}{b}}$. To incorporate the periodicity of the environment (e.g. food supplies, seasonal effects of weather, mating habits, etc.) in many biological and ecological dynamic systems, it is necessary to consider periodicity of the parameters in the models.

Many authors have considered the predator-prey model with Holling type IV functional response by assuming a periodic environment. However, there is no published paper considering the neutral impulsive system with *nonmonotonic* responses. Thus, we consider a neutral impulsive differential equation. An impulsive predator-prey model with non-monotonic functional response is investigated. Some novel sufficient conditions are obtained for the *nonexistence* of periodic solutions and the global existence of *at least one or two* positive periodic solutions. Our method is based on Mawhin's coincidence degree and novel estimation techniques for a priori bound of unknown solutions. An application is presented to illustrate the feasibility and effectiveness of our main results.

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Sliding Vectors on codimension-2 Intersections of Discontinuity Sets in Piecewise Smooth Systems

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The aim of this work is to study the dynamics on the codimension-2 intersection of discontinuity sets of piecewise smooth dynamical systems in $\mathbb{R}^n$, $n \geq 3$. The questions posed concern the **natural extension of the Filippov Convention** and the **definition, existence & multiplicity of sliding vector fields**.

In [1] it is stated that, until then, no simple criteria were known for determining a priori whether and how many candidate sliding vectors exist. The following year, in [2], a classification of the dynamics on the intersection was demonstrated, describing possible behaviors. In both references, calculations on the problem at hand are suggested, and no general conditions indicating the existence and number of sliding vectors are posed, e.g. in terms of the orientation and magnitude of the extensions of the smooth vector fields on the intersection.

In the present work, the system is transcribed into a Slow-Fast setup with the use of Sotomayor-Teixeira regularization functions, as in [3]. **For the first time, the nature of the induced parametric surface is studied** (a hyperbolic paraboloid, called "canopy" in [1]). Then, **simple geometric criteria** are introduced in order to investigate **whether and how many sliding candidates exist (zero, one or two)**. **Their stability is classified**, and it is shown that even in the case where two candidates exist, one of them is stable and the other is unstable, thus **the sliding is in practice uniformly defined**.

For a generic case, **the existence of periodic orbits on the intersection is shown**. These orbits correspond to limit cycles emanating from a Hopf bifurcation of the regularized system. A **canard explosion** accompanying this Hopf bifurcation is calculated as indicated in [4], and confirmed computationally using the `auto-07p` software. For appropriate return mechanisms, this canard explosion leads to larger **relaxation oscillations**.

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Weak Quantum Chaos

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Recently, there has been a renewed interest in quantum chaos, this time not coming from its primary community but rather from the high energy theory community. It is widely accepted that quantum chaos can only be rigorously defined for systems with a semi-classical limit. Based on this observation, the best tool to quantify chaoticity of quantum systems we have had has been random matrix analysis - the spectra of the quantum systems whose semi-classical counterparts were chaotic could always be well described using ensembles of random matrices. As there is no notion of trajectories in quantum physics, traditionally there has not been a truly quantum dynamical quantifier of chaos.

Motivated by the studies of mixing (or scrambling, as they call it) of information in strongly gravitating quantum systems, it was proposed a couple of years ago to study chaos of many-body quantum systems using *Out-of-time-ordered correlation functions (OTOC)*:

$$C\,(x,t) = -\langle [w(x,t), v(0,0)]^2 \rangle_\beta$$

Here, w and v are local quantum operators acting at different positions and times. Notice that taking the square of the commutator gives terms which are not time-ordered, hence the name of the object. The concept was proposed based on a work of Larkin and Ovchinnikov from 1969, where the OTOC was connected to the Lyapunov exponent of the trajectories of electrons scattered by impurities in a superconductor. It could be the first genuinely quantum quantifier of chaos.

In a recent work with Tomaž Prosen and Sašo Grozdanov, we have shown that the OTOC as defined above does not make much sense for the big majority of condensed matter systems. The OTOC in this case is bounded and hence its dynamics always only a transient effect, so it cannot be used to measure chaos. Instead, we have proposed to measure chaos in these systems in terms of the density of OTOC of extensive observables (dOTOC). Such an object has the required properties to be a well defined quantifier of chaos. We have shown that the growth of dOTOC in systems with local interaction is always subexponential. But, because such systems can still have chaotic semi classical limits, we have proposed to call them weakly quantum chaotic. Based on the example of the Kicked quantum Ising model, we have demonstrated that dOTOC is very sensitive to the interaction strength and can distinguish integrable from nonintegrable systems.

The topic of OTOC is interesting both from its physical content and the aspect of social dynamics of scientific communities.

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Aspects of diffusion in the stadium billiard

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Dynamical billiards are valuable model systems in the study of both classical and quantum chaos. A classical dynamical billiard consists of a point particle inside a prescribed region bounded by walls, where the particle is specularly reflected. By varying the shape of the boundary anything from an integrable to a mixed type and even an ergodic, fully chaotic, Hamiltonian system can be achieved.

In this report I will present some very recent results on the diffusion in the stadium billiard introduced by Bunimovich. The stadium billiard is proven to be rigorously ergodic and mixing. I will show that the results for the diffusion in momentum space obtained by numerical calculations of the stadium dynamics agree very well with an inhomogeneous diffusion equation. The diffusion constant is a parabolic function of the canonical momentum.

The model enables us to extract the classical transport time, an important parameter in the study of localization of chaotic eigenstates in the quantum stadium billiard.

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Dynamics of a subthalamic nucleus-globus palidus network with three delays

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We analyse a model of the subthalamic nucleus (STN)-globus palidus (GP) network with three different transmission delays. A time-shift transformation reduces the model to a system with two time delays, for which the existence of a unique steady state is established. Conditions for stability of the steady state are derived in terms of system parameters and time delays. Numerical stability analysis is performed using traceDDE to investigate different dynamical regimes in the STN-GP model, and to obtain critical stability boundaries separating stable (healthy) and oscillatory (Parkinsonian-like) neural firing. Direct numerical simulations of the fully nonlinear system are performed to confirm analytical findings, and illustrate different dynamical behaviours of the system.

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Ferromagnetic Potts models with multi site interaction

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The Potts model has been widely explored in the literature for the last few decades. While many analytical and numerical results concern with the traditional two site interaction model in various geometries and dimensions, little is yet known about models where more than two spins simultaneously interact. We consider a Ferromagnetic Four site interaction Potts model on the Square lattice (FFPS), where the four spins reside in the corners of an elementary square. Each spin can take an integer value $1, 2, ..., q$. We write the partition function as a sum over clusters consisting of monochromatic faces. When the number of faces becomes large, tracing out spin configurations is equivalent to enumerating large lattice animals. Based on this observation, systems with $q < 4$ and $q > 4$ exhibit a second and first order phase transitions, respectively. The transition nature of the $q = 4$ case is borderline. Indeed, if contributions from lattice animals at criticality are exclusive, it is second order. Nevertheless, other contributions can make it first order. For any q, a critical giant component (GC) is formed. In the first order case GC is simple, while it is fractal when the transition is continuous. Using simple arguments, and assuming (to leading order) the number of faces and sited in GC are equal, we obtain a (zero order) bound on the transition point. It is argued that this bound should apply for other lattices as well. Next, taking into account higher order sites contributions, the critical bound becomes tighter. Moreover, for $q > 4$, if corrections due to contributions from small clusters are negligible in the thermodynamic limit, the improved bound should be exact. Our analytical predictions are confirmed by an extensive numerical study of FFPS, using the Wang-Landau method. In particular, the $q = 4$ marginal case is supported by a very ambiguous pseudo-critical finite size behaviour.

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Global dynamics of a mechanical system with dry friction

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Friction is a common phenomenon in nature. When two surfaces come into contact, frictional forces are produced by each surface on the other. In this talk, the global dynamics of a mechanical system with dry friction is completely analyzed. This class of discontinuous and transcendental piecewise differential system can exhibit rich and complicated dynamical phenomena, such as Hopf bifurcation, grazing bifurcation, grazing-sliding bifurcation and bifurcations of limit cycles. At last all global phase portraits of the system are presented on the Poincaré disc. Moreover, we illustrate our results by some numerical examples.

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Approximating generic KAM systems with sequences of systems with sharply divided phase space

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New approach for construction of Hamiltonian systems with a finite number of KAM islands will be demonstrated. The approach allows to approximate a given Hamiltonian system with a divided phase space and with an infinite number of families of KAM tori (KAM islands in 2D) by the sequence of Hamiltonian systems with increasing numbers of KAM islands. Important feature of the algorithm is that each island in any of the approximating systems is a (sub)island presented in the initial Hamiltonian system with an infinite number of islands. In order to achieve this some parts of the phase space were cut out. The remaining part is phase space of the approximating system. By cutting out smaller and smaller parts of the phase space of the system under study we get approximating Hamiltonian systems which have more and more islands.

The main idea of the procedure presented is based on the fact that in a typical Hamiltonian system each family of the KAM tori is surrounded by other KAM families corresponding to higher resonances. The pieces, which were cut out from the phase space, contain at least one island from any of the island chains corresponding to higher resonances. Therefore only a (finite) number of KAM islands corresponding to lower resonaces remain.

The idea is demonstrated in a sequence of billiards, which is formed by cutting the lemon billiard with straght-line segments in such a way that the periodic orbits of the original billiard are added one by one. All the systems constructed in such a way have a sharply divided phase space.

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CONCERT BY JURE GORUČAN, PIANO

Wednesday 6 September 2017 at 19:00, Villa del Grumello, Como

Frederic Chopin:

Piano Sonata No.2, Op.35

I. Grave, Doppio Movimento
II. Scherzo
III. Marche Funebre
IV. Finale: Presto

Rastko Buljancevic:

Prelude and Fugue in b-minor
Toccata Histerica and Fugue

5-10 min break

Maurice Ravel:

Gaspard de la Nuit

I.Ondine
II. Le Gibet
III. Scarbo

Franz Liszt:

Mephisto Waltz No.1, S.514

FESTIVE CONFERENCE DINNER

Wednesday 6 September 2017 at 20:00, in Villa del Grumello, Como